

The author proposes a clever experiment and investigates what we can learn from it. Unfortunately, he draws an incorrect conclusion and therefore misses a conclusion that is both correct and interesting.

I.

The incorrect conclusion is that this experiment can be used to establish a (non-conventional) one-way speed of light, which is well-known to be impossible. When one's reasoning leads to a conclusion that one can do the impossible, the right response is to work through the impossibility proof step by step, applying each step to one's own argument, and thereby pinpoint one's mistake. Unfortunately, the author apparently hasn't bothered to do this.

The experiment is this: Imagine two locations A and B , say 2 miles apart (and with B to the right of A), with C at the midpoint. Standing at A , fire simultaneously a light beam toward B and a cannonball toward C . The arrival of the lightbeam at C triggers the firing of another cannonball toward C .

Let v_R and v_L be the rightward and leftward velocities of the cannonball, and c_R the rightward velocity of light. Then the difference in the arrival times of the cannonballs at C is easily seen to be

$$T = 2/c_R + 1/v_L - 1/v_R \quad (1)$$

From this we can infer that

$$c_R = \frac{2v_L v_R}{v_L - v_R + T v_L v_R} \quad (2)$$

We can define the "standard theory" to be that $v_R = v_L$ and $c_R = 1$ (taking the two-way speed of light to be 1); that is, velocities don't depend on directions. So according to the standard theory, the above equation reduces to

$$1 = 2/T \quad (3)$$

and therefore predicts that $T = 2$.

However, the implication goes in only one direction. The standard theory predicts that $T = 2$, but the observation that $T = 2$ does not imply the standard theory.

II.

It is easy to measure the two-way speed of a cannonball by firing it toward a target a known distance away, having it bounce back perfectly inelastically, and observe the time needed for the round trip. From this one infers the average velocity. Call it v . Now v_L and v_R (which are not observed independently) must satisfy the equation

$$1/v_L + 1/v_R = 2/v$$

or

$$v_R = \frac{v v_L}{2v_L - v} \quad (4)$$

Plugging equation (4) into equation (3) gives

$$c_R = \frac{v v_L}{2v_L + v(T v_L - 2)} \quad (5)$$

and continuing to assume that $T = 2$ this becomes

$$c_R = \frac{v v_L}{v_L + v(v_L - 1)} \quad (6)$$

From this we deduce:

Theorem. If $v_L = v$ (that is, if cannonballs travel at the same speed in both directions) then $c_R = 1$ (that is, light travels at the same speed in both directions).

But if $v_L \neq v$, equation (6) seems to imply that c_R depends not just on the arbitrarily chosen v_L but also on the observed value of v . This in turn suggests that if we were to repeat the experiment with a different

type of cannon or a different type of cannonball, we would get a different value for c_R — unless in fact the standard theory is true. Thus we seem to have an experimental way of distinguishing the standard theory from alternatives after all.

We will see that this is an illusion.

III.

It is very true that the argument at the end of Section II eliminates a great many alternative theories. But it does not eliminate all of them. Consider a theory in which $v_L = 1/(1 + Av)$ and $v_R = 1/(1 - Av)$ for some constant $A \in (-1, 1)$. Then equation (6) becomes

$$c_R = \frac{1}{1 - A} \quad (7)$$

and the dependence on v is eliminated.

To conclude, we have the following theorem:

Theorem. If v_L and v_R are chosen as in the preceding paragraph, and if the constant A is applied consistently to compute one-way speeds for cannonballs of all sorts, then the theory, combined with the experimental observation that $T = 2$, is consistent with any value of c_R in the interval $(1/2, \infty)$.

Thus the theory, together with the experimental result, reveals nothing about the one-way speed of light that is not already obvious.

IV.

However, we did learn something: If the experiment is to be repeated with multiple cannonballs, it's not enough to choose v_L and v_R in any arbitrary way that yields the observed average velocity v .

It *is* however enough to choose them according to the scheme in the first paragraph of Section III. It turns out (and here is the interesting implication that the author missed) that this implication goes both ways:

Theorem. In order for all versions of the experiment (i.e. versions with different speed cannonballs) to yield the same value for c_R , one must consistently choose the values of v_L and v_R according to the scheme $v_L = 1/(1 + Av)$, $v_R = 1/(1 - Av)$ for some *fixed* constant $A \in (-1, 1)$.

Sketch of proof: Write v_L as a function of v and differentiate the right hand side of equation (6). In order for that expression to be independent of v , we need this derivative to be equal to zero. That gives a differential equation for v_L , which has the advertised set of solutions.